

Complex Number

4-10/2015

II Repetition

MMR
M.r

a

$$x^2 - 9 = (x-3)(x+3)$$

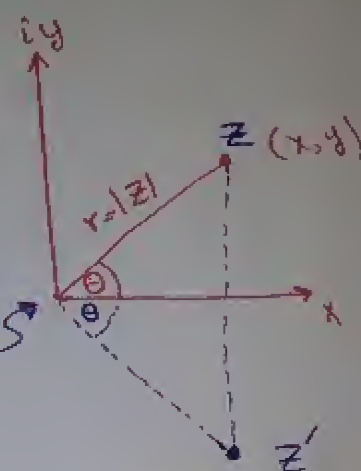
$$x^2 + 9 = (x-3i)(x+3i)$$

$$Z = x + iy = re^{i\theta} = r[\cos\theta + i\sin\theta]$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\operatorname{Re}(Z) = x$$

$$\operatorname{Im}(Z) = y$$



Conjugate المرافق

$$\bar{Z} = x - iy = re^{-i\theta}$$

$$(\overline{w + Z}) = \bar{w} + \bar{Z}$$

$$(\overline{wZ}) = \bar{w} \cdot \bar{Z}$$

$$\overline{\left(\frac{w}{Z}\right)} = \frac{\bar{w}}{\bar{Z}}$$

modulus المقياس

هو بعد النقطة عن نقطة الاصل

$$r = |Z| = \sqrt{x^2 + y^2}$$

18) Properties

$$① |z_1 + z_2| \leq |z_1| + |z_2|$$

$$② |z_1 + z_2| \geq |z_1| - |z_2|$$

$$③ |z_1 z_2| = |z_1| \cdot |z_2|$$

$$④ \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$$

$$⑤ |z^n| = |z|^n$$

$$⑥ |z|^2 = z \cdot \bar{z}$$

argument ~~الزاوية~~

$$\theta = \arg(z) = \tan^{-1} \left(\frac{y}{x} \right)$$

في الزاوية المعصورة بين النقطة ونقطة الأصل ~~في محور السينات~~

Properties

$$① \arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$$

$$② \arg(z_1 / z_2) = \arg(z_1) - \arg(z_2)$$

$$③ \arg(z)^n = n \arg(z)$$

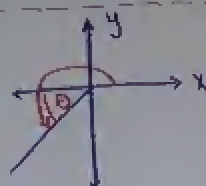
(c)



$$y + i x - (\pi - |\theta|)$$



$$y + i x + |\theta|$$



$$y - i x - (\pi + |\theta|)$$



$$y - i x + (2\pi - |\theta|)$$

Adding & subtracting.

$$Z_1 \pm Z_2 = (x_1 \pm x_2) + i(y_1 \pm y_2)$$

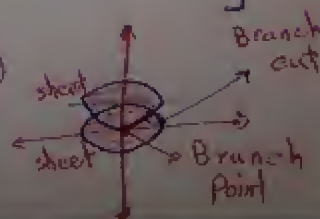
Roots of complex number

$$Z^{1/n} = r^{1/n} \left[\cos\left(\frac{\theta + 2\pi k}{n}\right) + i \sin\left(\frac{\theta + 2\pi k}{n}\right) \right]$$

$n=2 \rightarrow$ square root

$k = (0, 1, 2, \dots, n-1)$

$n \rightarrow$ degree of root



(d)

Sheet 1

~~Find the real and imaginary~~

★ Find modulus & argument of .

$$(1) \quad z = \frac{2+i}{3+4i}$$

$$|z| = \left| \frac{2+i}{3+4i} \right| = \frac{|2+i|}{|3+4i|} = \frac{\sqrt{4+1}}{\sqrt{9+16}} = \frac{1}{\sqrt{5}}$$

$$\arg(z) = \arg\left(\frac{2+i}{3+4i}\right) = \arg(2+i) - \arg(3+4i)$$

$$\arg(z) = \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{4}{3}\right)$$

$$\arg(z) = -26.565$$

لأنه يغير قيم الزوايا

$$(2) \quad \frac{1+2i}{3-4i} + \frac{2-i}{5i}$$

$$|z| = \left| \frac{1+2i}{3-4i} + \frac{2-i}{5i} \right| = \left| \frac{5i(1+2i) + (3-4i)(2-i)}{(3-4i)(5i)} \right|$$

$$|z| = \left| \frac{5i-10+6-3i-8i-4}{15i+20} \right| = \left| \frac{-8-6i}{20+15i} \right|$$

$$|z| = \frac{|-8-6i|}{|20+15i|} = \frac{\sqrt{64+36}}{\sqrt{(20)^2+(15)^2}} = 0.4$$

$$\arg \left(\frac{-8-6i}{20+15i} \right) = \arg(-8-6i) - \arg(20+15i)$$

الجزء الخالي، الجزء الخالي

$$\arg(z) = \left(\pi + \tan^{-1}\left(\frac{6}{8}\right) \right) - \tan^{-1}\left(\frac{15}{20}\right) = \underline{\underline{\pi}}$$

Find real & imagine part = find polar form

$$z = x+iy = r [\cos \theta + i \sin \theta]$$

$$\boxed{a} \quad (1+\sqrt{3}i)^6$$

$$r = |z| = |(1+\sqrt{3}i)|^6 = |(1+\sqrt{3}i)|^6$$

$$= (\sqrt{1+3})^6 = \underline{\underline{2^6}}$$

$$\theta = \arg(z) = \arg(1+\sqrt{3}i)^6 = 6 \arg(1+\sqrt{3}i)$$

الجزء الخالي

$$= 6 + \tan^{-1} \frac{\sqrt{3}}{1} = \underline{\underline{2\pi}}$$

$$\therefore z = 2^6 [\cos 2\pi + i \sin 2\pi] = 2^6 [1 + i0]$$

$$z = \underbrace{2^6}_{\text{real part}} + \underbrace{0i}_{\text{Im part}}$$

$$16) \left(\frac{1+i}{1-i} \right)^4$$

$$r = |z| = \left| \frac{1+i}{1-i} \right|^4 = \left(\frac{|1+i|}{|1-i|} \right)^4 = \frac{1}{1}$$

$$\theta = \arg(z) = \arg \left(\frac{1+i}{1-i} \right)^4 = 4 [\arg(1+i) - \arg(1-i)]$$

$$\theta = 4 [\tan^{-1}(1) - (2\pi - \tan^{-1}(1))] = \underline{\underline{-6\pi}}$$

$$\therefore z = \cos(-6\pi) + i \sin(-6\pi)$$

$$z = 1 + 0i$$

2) show that

$$\begin{aligned} \text{[a]} \quad & 1 + \cos(\theta) + \cos(2\theta) + \dots + \cos(n\theta) \\ &= \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2 \sin(\frac{\theta}{2})} \end{aligned}$$

$$\underline{e^{i\theta}} = \underline{\cos \theta} + i \sin \theta$$

$$\therefore \cos \theta = \underline{\text{Re}(e^{i\theta})}$$

$$\underline{\text{L.H.S}} \rightarrow \text{Re} \left[1 + e^{i\theta} + e^{i2\theta} + \dots + e^{in\theta} \right]$$

9

المجموع الهندسي
 $a + ar + ar^2 + \dots = a \frac{1 - r^{n+1}}{1 - r}$

$$\operatorname{Re} \left[\frac{1 - e^{i(n+1)\theta}}{1 - e^{i\theta}} \right] \times \frac{e^{-i\frac{\theta}{2}}}{e^{-i\frac{\theta}{2}}} \quad \text{المضروب}$$

$$= \operatorname{Re} \left[\frac{e^{-i\frac{\theta}{2}} - e^{i(n+\frac{1}{2})\theta}}{e^{-i\frac{\theta}{2}} - e^{i\frac{\theta}{2}}} \right]$$

$$= \operatorname{Re} \left[\frac{\cos(-\frac{\theta}{2}) + i \sin(-\frac{\theta}{2}) - \cos(n+\frac{1}{2})\theta - i \sin(n+\frac{1}{2})\theta}{\cos(-\frac{\theta}{2}) + i \sin(-\frac{\theta}{2}) - \cos(\frac{\theta}{2}) - i \sin(\frac{\theta}{2})} \right]$$

$$= \operatorname{Re} \left[\frac{\cos(\frac{\theta}{2}) - i \sin(\frac{\theta}{2}) - \cos(n+\frac{1}{2})\theta - i \sin(n+\frac{1}{2})\theta}{-2i \sin(\frac{\theta}{2})} \right]$$

$$= \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\theta}{2 \sin(\frac{\theta}{2})} = \text{R.H.S} \quad \times$$

11) show that

$$(b) \frac{(\cos 2\theta - i \sin 2\theta)^7 (\cos 3\theta + i \sin 3\theta)^{-5}}{(\cos 4\theta + i \sin 4\theta)^{12} (\cos 5\theta - i \sin 5\theta)^{-6}}$$

$$= \cos 107\theta - i \sin 107\theta$$

$$\text{L.H.S} = \frac{(e^{-i2\theta})^7 (e^{i3\theta})^5}{(e^{i4\theta})^{12} (e^{-i5\theta})^{-6}} \quad \text{Soln.}$$

$$= \frac{e^{-i14\theta} \cdot e^{-i15\theta}}{e^{i48\theta} \cdot e^{i30\theta}} = e^{i\theta[-14-15-48-30]}$$

$$= e^{-i107\theta} = \cos(107\theta) - i \sin(107\theta) = \text{R.H.S}$$

14) show that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$$

$$\therefore |z|^2 = z \cdot \bar{z}$$

$$\text{L.H.S} \rightarrow (z_1 + z_2)(\overline{z_1 + z_2}) + (z_1 - z_2)(\overline{z_1 - z_2})$$

$$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) + (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 + z_1\bar{z}_1 - z_1\bar{z}_2 - z_2\bar{z}_1 + z_2\bar{z}_2$$

$$= 2z_1\bar{z}_1 + 2z_2\bar{z}_2 = 2|z_1|^2 + 2|z_2|^2 = \text{R.H.S}$$

5] Use De Moivre theorem to obtain

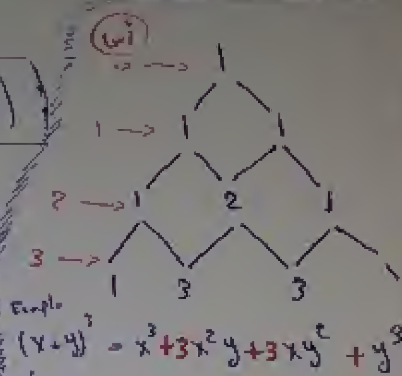
$\cos 3\theta$ & $\frac{\sin 3\theta}{\sin \theta}$ in terms of power of $\cos \theta$.

De Moivre theorem

$$(\cos \theta + i \sin \theta)^n = (\cos n\theta + i \sin n\theta)$$

at $n=3$

$$(\cos \theta + i \sin \theta)^3 = (\cos 3\theta + i \sin 3\theta)$$



$$= (\cos \theta)^3 + 3(\cos \theta)^2(i \sin \theta) + 3(\cos \theta)(i \sin \theta)^2 + (i \sin \theta)^3$$

$$= \cos^3 \theta + 3i \cos^2 \theta \sin \theta + -3 \cos \theta \sin^2 \theta - i \sin^3 \theta$$

Real Part = $\cos 3\theta + i \sin 3\theta$

$$\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta$$

$$\cos 3\theta = \cos^3 \theta - 3 \cos \theta \cdot [1 - \cos^2 \theta]$$

Im Part

$$\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$$

$$\frac{\sin 3\theta}{\sin \theta} = 3 \cos^2 \theta - (1 - \cos^2 \theta)$$